PHYS 4330 Theoretical Mechanics

Homework # 4

Submission deadline: 6 February 2024 at 11:59 pm Eastern Time

Submission Instructions: Homework is submitted on Gradescope to Homework 4.

IMPORTANT NOTE: *Only 2* of these problems will be graded for credit (at 10 points each)!! We will not disclose which problems are being graded before the deadline. This means that you will need to submit completed answers to all questions or risk getting a 0 if we grade a problem you didn't do.

1. (a) Write down the Lagrangian $L(x_1, x_2, \dot{x}_1, \dot{x}_2)$ for two particles of equal masses $m_1 = m_2 = m$ confined to the x-axis and connected by a spring with potential energy $U = \frac{1}{2}k(x_1 - x_2)^2$. *[Ignore gravity and friction.]*

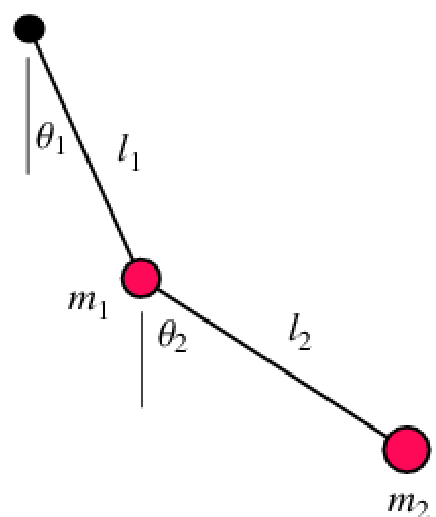
Calculate Lagrange's diff. equations for each mass, i.e. $\frac{d}{dt}\frac{\partial L}{\partial \dot{x}_i} - \frac{\partial L}{\partial x_i} = 0$ for i = 1,2.

(b) Rewrite the Lagrangian L in terms of new variables $X_{CM} = \frac{1}{2}(x_1 + x_2)$ (the center-of-mass position) and $x = x_1 - x_2$ (relative distance between mass 1 and 2). The mass 1 remains at all times to the right of mass 2.

(c) Calculate Lagrange's equations of motion of X_{CM} and x. Solve the differential equations for X_{CM}(t) and x(t) and describe the motion (using full sentences).

(d) Consider the differential equations for x_1 and x_2 and compare them to the differential equations obtained for X_{CM} and x. What is gained by introducing X_{CM} and x?

2. Consider a double pendulum made of two masses, m_1 and m_2, and two massless rods of lengths l_1 and l_2, as shown in the Figure *[assume gravitational acceleration points down the page]*. The black point at the top of the Figure is a stationary anchor point.



(a) Find the Lagrangian for the system in terms of θ_1 and θ_2.

(b) Now, simplify your Lagrangian using the assumption that the masses are identical and the lengths of the rods are identical. ($m_1 = m_2 = m$ and $l_1 = l_2 = l$) Using this simplified Lagrangian find the differential equations for θ_1 and θ_2.

(c) Simplify your equations of motion for the case of small angle oscillations. To do this keep only the leading terms of the approximations.
*[Hint: if x << 1 and y << 1, then x*y <<<<1 and can therefore be ignored]*
You do not need to solve the differential equations for this problem.

3. A block of mass m is held motionless on a frictionless plane of mass M and an angle of inclination θ. The plane rests on a frictionless horizontal surface. The block is released from rest. Find the Lagrangian L of the block and the plane and Lagrange's equations of motion. Solve for the accelerations of the blocks in terms of g, m, M, and θ.

[You can either use the coordinates x_1 and x_2 that are shown in the image or you can define a new two dimensional coordinate for the block and ramp. Just be clear on what coordinates you are using.]