PHYS 4330 Theoretical Mechanics

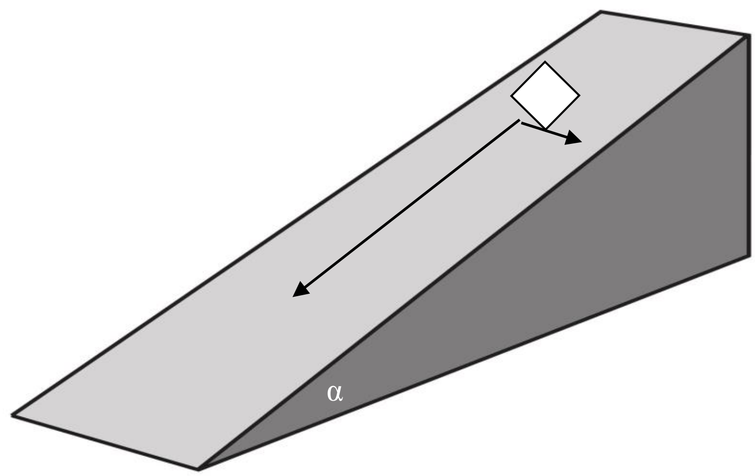
Homework # 3

Submission deadline: 30 January 2024 at 11:59 pm Eastern Time

Submission Instructions: Homework is submitted on Gradescope to Homework 3.

1. Consider a box of mass *m* moving on a frictionless plane that forms an angle α with the horizontal. Assume that the box starts from rest on the inclined plane. Define one coordinate to point directly down the incline (with the slope) and a second coordinate that is measured horizontally across the slope (as shown in the image below). Make sure to clearly label each of your axes. (you can ignore the 3^{rd} dimension which would be "up" from the face of the plane)

(a) Write down the Lagrangian for the system.
(b) Find and solve Lagrange's equation of motion for both coordinates. [x(t)=, y(t)=]
(c) Redo the entire problem using Newtonian Mechanics (Newton's Second Law) and show that you get the same equations of motion for each coordinate.



[10 points]

2. Calculate the kinetic energy $T = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right)$ of a free particle of mass *m* in:

(a) cylindrical coordinates and find Lagrange's equations of motion in cylindrical coordinates.
(b) spherical coordinates and find Lagrange's equations of motion in spherical coordinates.

You need to start from the Kinetic energy in Cartesian coordinates and the coordinate transform for the given new coordinate system and then show your work to get the new kinetic energies.

You do not need to solve the differential equation.

[10 points]

3. The Figure is a top down view of a smooth wire hoop on a horizontal surface that is forced to rotate at a fixed angular velocity ω about a vertical axis through the point *A.* A bead of mass *m* is threaded on the hoop and is free to move around it, with its position specified by the angle ϕ that it makes at the center with the diameter AB.

(a) Find the Lagrangian for this system using ϕ as your generalized coordinate.

(b) Use the Lagrange equation of motion to show that the bead oscillates about point B exactly like a simple pendulum.

(c) What is the frequency of these oscillations if their amplitude is small?

[10 points]