PHYS 4330 Theoretical Mechanics

Homework # 1

Submission deadline: 16 January 2024 at 11:59 pm Eastern Time

Submission Instructions: Homework is submitted on Gradescope to Homework 1.

1. Consider a projectile fired from the origin of a three-dimensional coordinate system on Earth (+z is the "up" direction). Assume that the projectile is fired with an initial velocity v_0 at an angle α above the horizontal in the direction of the +x axis

(a) Calculate the angular momentum $\vec{L} = \vec{r} \times \vec{p}$ of the projectile. $\vec{r}$ and $\vec{p}$ are measured with respect to the origin of the coordinate system.

(b) Calculate the torque due to gravity $\vec{N} = \vec{r} \times \vec{F}_g$ acting on the projectile with respect to the origin of the coordinate system.

(c) Demonstrate that the torque is equal to the time rate of change of the angular momentum.

[10 points]

2. A block of mass *m* slides down a frictionless incline (Figure 2). The block is released at height *h*. The bottom of the loop should be treated as a circular segment with radius *R*.
(a) What is the force of the inclined track on the block at the bottom (point A)?
(b) At what speed does the block leave the track?
(c) How far away from point A does the block land on level ground? Give your answer in terms of *R, h, m,* and *g. (Hint: Set Point A to be the origin of your coordinate system)*
(d) Sketch the potential energy *U(x)* of the block. Indicate the total energy on the sketch.

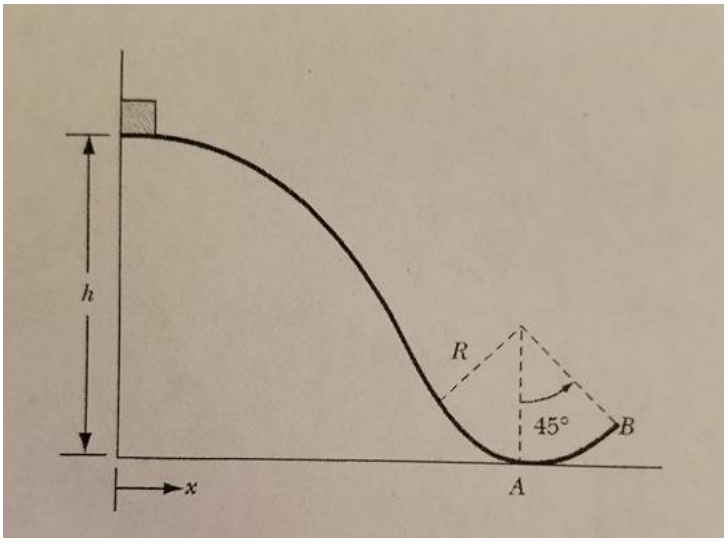

Figure 2

[10 points]

3. Consider a projectile of mass *m* fired vertically upward in a constant gravitational field ***g***. The direction of the gravitational field is downward. The initial velocity of the projectile is $v(0) = v_0$. Calculate the time, t_m, required for the projectile to reach maximum height for the case of

(a) zero resisting force: $F_r = 0$.

(b) a resisting force proportional to the square of the velocity of the projectile: $F_r = - c\, v^2$, with *c* being a positive constant.

[10 points]

4. A particle of mass *m* has speed $v(x) = \frac{\alpha}{x}$.

(a) Calculate the force *F (x)* responsible.

(b) Calculate the displacement *x(t)* of the particle.

(c) Calculate the speed of the particle *v(t)* as a function of time.

(d) After having calculated *x(t)* and *v(t)* check whether the solutions reproduce the given *v(x)*.

(e) Calculate the force *F (t)=m (dv/dt)* as a function of time *t*. Is the result in agreement with the solutions to (a) and (b)?

Assume *v(x=0)=0* and *x (t=0)=0*.

[10 points]